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Research Article

Intermittent Plurisink Model and the Emergence of Complex Heterogeneity Patterns: A Simple Paradigm for Explaining Complexity in Soil Chemical Distributions

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The spatial complexity of the distribution of organic matter, chemicals, nutrients, and pollutants has been demonstrated to have multifractal nature. This fact supports the possibility of existence of some emergent heterogeneity structure built under the evolution of the system. The aim of this paper is providing a consistent explanation of the mentioned results via an extremely simple model.

1. Introduction: Searching Explanations for Soil Heterogeneity

Heterogeneity and complexity are ubiquitous at all scales in soil and hydrologic systems. Nowadays, new technologies are of an invaluable help for providing a great number of highly calibrated field measurements. One can get a huge amount of data from computer tomography of soil samples at microscopic scales, digital terrain catchments of landscapes, and river basins among many other technological tools. Then mathematical tools are needed to analyse and interpret those data as well to construct models to predict. However, along the way needed to get such a final purpose, scientists also need to understand why the heterogeneity is produced and what the organizing principles that might underlie the heterogeneity and complexity are (McDonnell et al. [1]). Also it is encouraged to explore the scaling behaviour of heterogeneity and the emergent properties in soil and hydrologic systems. In this paper we are mainly interested in some aspects concerning the heterogeneity in the soil scenario.

Certainly we believe that the above understanding, besides providing coherence to science, also may be useful to get the practical purpose itself. In the case of soil, an illustrating example supporting this and the issues mentioned above is the study of soil texture heterogeneity. On one hand, Multifractal Analysis of fine granulometry soil data obtained by laser diffraction techniques provides information about the scaling behaviour of particle size distribution (PSD) heterogeneity (Montero [2]). In a second step, models able to replicate the heterogeneity formerly shown may be useful for prediction purposes (Martín and García-Gutiérrez [3]). The answer to why such heterogeneity exists, however, is not an easy issue since different sources of heterogeneity should be expected. In Frisch and Sornette [4] and Sornette [5], it is suggested that the fractal behaviour might be the result of a natural mixing of simple multiplicative process that takes place along the fragmentation of different particles, also pointing out that there is no accepted theoretical explanation. Recently fragmentation algorithms were proposed to replicate the multifractal nature of soil PSD (Martín et al. [6]). In this respect it is needed to say that any partial, but coherent, explanation should help to understand the (possibly several ones) organizing principles involved.

On the other hand, the spatial complexity of the distribution of organic matter, chemicals, nutrients, and pollutants has been studied by different authors (Kravchenko et al. [7], Lehmann et al. [8]). Multifractal Analysis has been successfully used to study the spatial variability of chemicals and organic matter contents, which is characterized by the generalized fractal dimensions (Kravchenko et al. [7]). Searching why such structured heterogeneity exists, a reducionist approach based in the description of transport equations in soil, seems an unlikely choice to describe the emergence of such complex pattern across the spatial scales. On the contrary, an explanation based on the fact that many complex systems in nature evolve in an intermittent burst-like way rather than in a smooth gradual manner (Rodríguez-Iturbe and Rinaldo [9]) would be more adequate. Further, such a kind of structured heterogeneity is commonly interpreted as the result of chaos or self-organization which leads to the emergent structure built under the evolution of the system (Sornette [5]). The aim of this paper is to provide a small contribution via an extremely simple model, which gives a consistent explanation to the mentioned results on spatial variability of chemicals or pollutants in soil.

The paper is organized as follows. In Section 2.1 the model is presented and in Section 2.2 the entropy scaling analysis method is described. Section 3 is devoted to analysing the results obtained in different simulations and their discussion.

2. Material and Methods

2.1. The Model. Let us suppose that S is a soil area square-shaped. Suppose further that at any of the four corners there is a sink i ($i = 1, 2, 3, 4$) randomly acting in an intermittent manner. Suppose each sink i acting with relative frecuency p_i. A pollutant deposit ("pollutant seed") is supposedly located in an arbitrary point of the square. When a given sink i acts, its suction action is able to attract the pollutant matter to another point reducing the distance to the sink in a factor $r_i < 1$, where the pollutant rests until a new (or the same) sink acts. This factor reflects the mean value of the suction power of the respective sinks. However, the "flying" pollutant matter leaves a unit of pollutant at any point where the pollutant "rests" along its travelling.

Although a much more sophisticated model might be constructed for a more realistic performance under the same essential idea, we rather prefer to emphasize how complexity may appear under quite simple and natural actions evolving in time.

2.2. Measuring Heterogeneity. When the model is implemented a first goal is applying mathematical tools in order to parameterize heterogeneity in a reliable manner.

For simplicity let us assume that the unit square S in the plane is the support of a distribution μ with highly heterogeneous features. In order to scrutinize its heterogeneity, let us consider a collection (mesh) of $2^k \times 2^k$ ε-boxes, $P_\varepsilon = \{R_i : i = 1, 2, \dots, 2^{2k}\}$, of side length $\varepsilon = 2^{-k}$, representing a partition of S for each value k, $k = 1, 2, 3, \dots$ (see Figure 1).

When the mass $\mu(R_i)$ inside any box R_i is known, the *Shannon entropy* (Shannon [10]) of μ with respect to a fix partition P_ε is given by

$$H_\mu(P_\varepsilon) = -\sum_{i=1}^{2^{2k}} \mu(R_i) \log \mu(R_i) \tag{1}$$

provided $\mu(R_i) \log \mu(R_i) = 0$ if $\mu(R_i) = 0$.

The number $H_\mu(P_\varepsilon)$ is expressed in information units (bits) and its extreme values are $\log 2^{2k}$, which corresponds to the most even (homogeneous) case—where all the squares have the same cumulative mass—and 0, which corresponds to the most uneven (heterogeneous) case—where the whole mass is concentrated in a single square. The Shannon entropy $H_\mu(P_\varepsilon)$ is a widely accepted measure of evenness or heterogeneity in the mass distribution μ at the scale level given by each partition P_ε. In fact, it can be shown that any measure of heterogeneity with the natural properties for such goal must be a multiple of $H_\mu(P_\varepsilon)$ (Khinchin [11]).

Using increasing values of k (decreasing values of ε) one can obtain an increasing amount of information about the distribution as $H_\mu(P_\varepsilon)$ grows to infinity. If such an increase is not erratic but rather conforms to a scaling or asymptotic behaviour of $H_\mu(P_\varepsilon)$ when $\varepsilon \downarrow 0$, then the *entropy or information dimension* of μ is defined (Rényi [12]) by means of the equation

$$D \approx \frac{-H_\mu(P_\varepsilon)}{\log \varepsilon}, \tag{2}$$

where "≈" means that $-H_\mu(P_\varepsilon)$ will linearly fit $\log \varepsilon$.

3. Results and Discussion

In order to implement the model, different set of values of r_i and p_i were selected ($i = 1, 2, 3, 4$). First close values of p_i were used under the assumption of similar intermittent frequencies, while the r_i values used try to investigate the effect of relative different suction powers. For any simulation the centre of the square has been chosen as initial position for the "pollutant seed." Then for any simulation a scaling entropy analysis has been made following Section 2.2.

Figures 2(a) and 2(b) show two different simulations of 500 points for the same p_i and r_i values ($p_1 = 0.29$, $p_2 = 0.21$, $p_3 = 0.29$, $p_4 = 0.21$, $r_1 = 0.7$, $r_2 = 0.5$, $r_3 = 0.7$, and $r_4 = 0.5$). The scaling analysis was made by using values $\varepsilon = 2^{-k}$ from $k = 1$ to $k = 6$. The mass $\mu(R_i)$ is given by the proportion of points inside any box R_i. The value of $H_\mu(P_\varepsilon)$ is plotted against $-\log \varepsilon$ and a linear fitting is implemented. The slope of the regression line gives an estimation of the entropy dimension with R^2 value as coefficient of determination. It can be noticed that the physical appearance of both simulations is quite different, thus illustrating the high influence of the random effect in this case. Also the scaling analysis reveals different results (D and R^2 values) for both simulated distributions. For the same p_i and r_i values, simulation of 20000 points leads to the results in Figures 3(a) and 3(b).

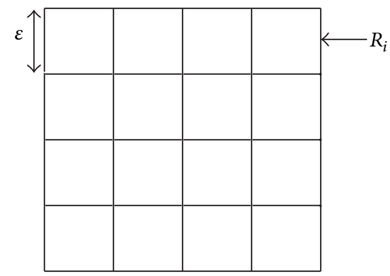
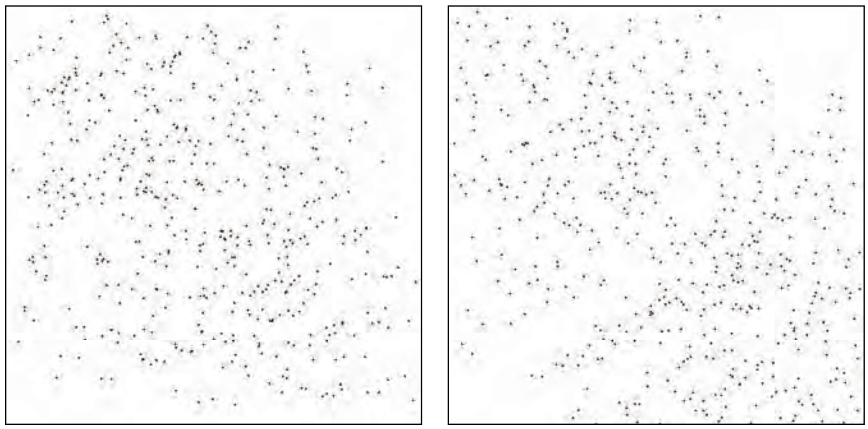


Figure 1: Partition of the support S by squares of side length ε.



(a) (b)

Figure 2: Two distributions generated with 500 points and the same probabilities and factors ($p_1 = 0.29$, $p_2 = 0.21$, $p_3 = 0.29$, $p_4 = 0.21$, $r_1 = 0.7$, $r_2 = 0.5$, $r_3 = 0.7$, and $r_4 = 0.5$).

Table 1

p_i	r_i	N	D	R^2
0,29-0,21-0,29-0,21	0,7-0,5-0,7-0,5	500	1,412	0,9645
			1,447	0,9578
		20000	1,937	0,9999
			1,936	0,9999

Table 2

Number of points N	D
5000	1,841
8000	1,873
10000	1,881
15000	1,903
20000	1,912
25000	1,913
30000	1,919
35000	1,921
40000	1,921
50000	1,923

Table 1 shows the results of this analysis. It is observed that the influence of the random component diminishes for increasing number of points used in the simulation. Also the

Table 3

p_i	r_i	D	R^2
0,25-0,25-0,25-0,25	0,5-0,5-0,5-0,5	1,991	0,9999
	0,7-0,7-0,5-0,5	1,951	0,9999
	0,8-0,6-0,7-0,6	1,924	0,9997

R^2 values become closer to 1. Figure 4 shows the value of the estimated entropy dimension for increasing number of points. Table 2 shows data involved in that figure.

Results clearly show the emergence of a mass distribution with a well-defined structured heterogeneity that the scaling analysis reveals. In fact the robustness of the results is based on a theorem of ergodic type (Elton [13]).

Finally Figures 5(a), 5(b), and 5(c) show the result of 20000 points simulation with the same probabilities $p_1 = p_2 = p_3 = p_4 = 0.25$ and different values of the factors r_i.

Table 3 shows the value of the estimated entropy dimension. The R^2 values obtained reflect the scale invariance of the resulting distributions.

Smaller r_i values representing greater suction powers have obvious influence on the heterogeneity of the final distribution which remains parameterized by the entropy dimension. In an intuitive sense, the entropy dimension value may be interpreted as uncertainty degree. In fact it can be used together with other parameters in interpolation

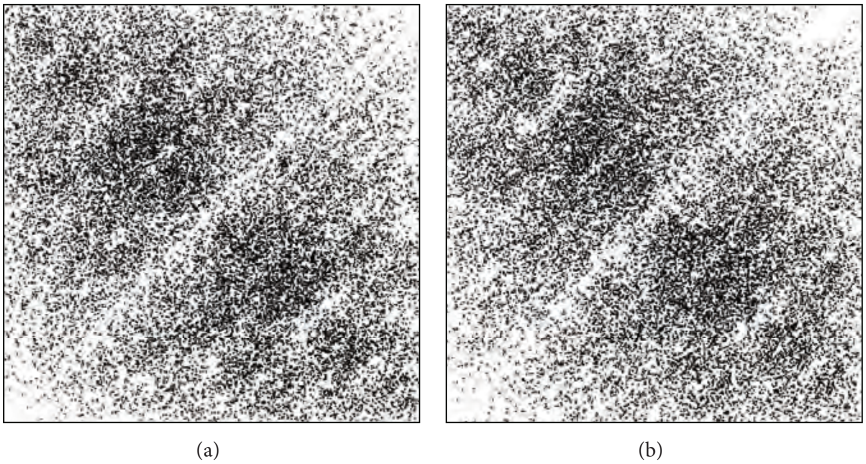


Figure 3: Two distributions generated with 20000 points and the same probabilities and factors ($p_1 = 0.29$, $p_2 = 0.21$, $p_3 = 0.29$, $p_4 = 0.21$, $r_1 = 0.7$, $r_2 = 0.5$, $r_3 = 0.7$, and $r_4 = 0.5$).

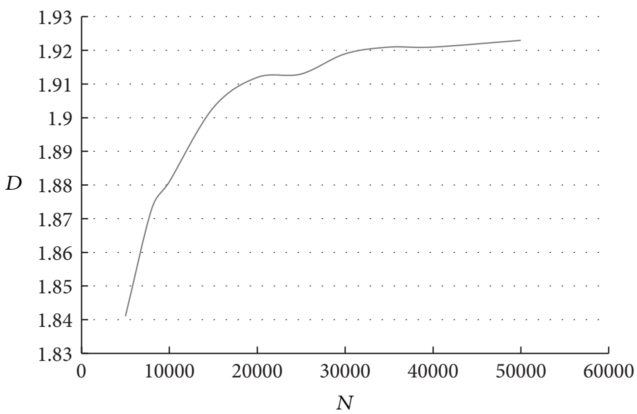


Figure 4: Value of the estimated entropy dimension D for increasing number of points N.

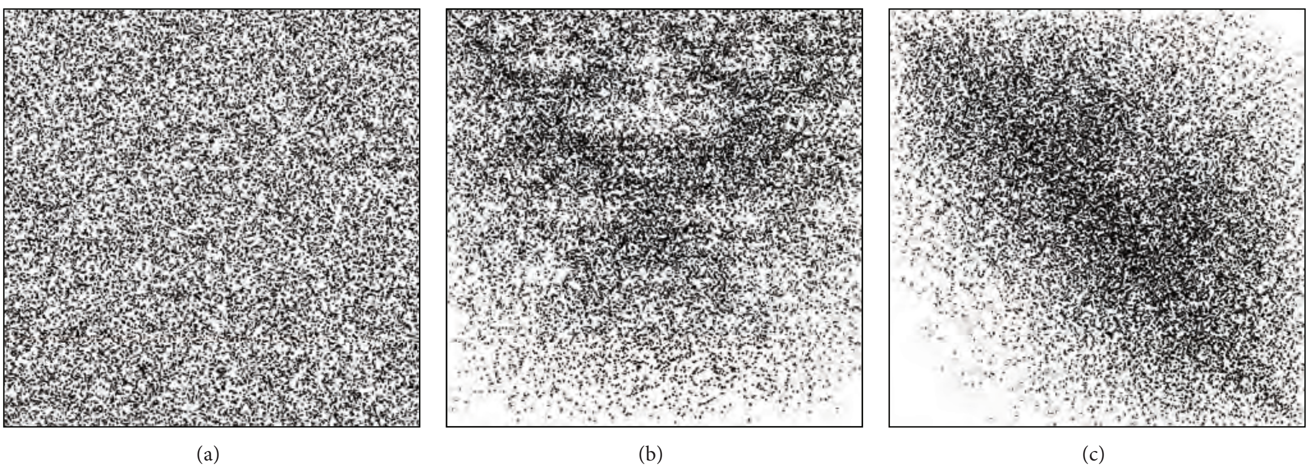


Figure 5: Distributions generated with 20000 points, the same probabilities, and different factors.

procedures in soil spatial variability studies (Kravchenko et al. [7]).

4. Conclusions

Heterogeneity is ubiquitous in many soil scenarios. In particular the spatial complexity of the distribution of organic matter, chemicals, nutrients, and pollutants is a frequent ingredient, which is in the focus of soil studies.

The understanding of why the heterogeneity is produced, and what the nature of such heterogeneity is, is a need under the scientific and practical points of view. Any coherent explanation on the origin of heterogeneity should help to understand it and to choose the adequate mathematical techniques for handling it with prediction purposes.

In this paper an extremely simple model is presented, which gives a consistent explanation of the complexity of spatial variability of chemicals or pollutants in soil shown in former studies.

The results shown here strongly suggest the use of scaling methods coming from fractal geometry for the study of this kind of distributions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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